

Take Flight With Understanding Numeration Plus[©] and Understanding Math Plus[©]

Fraser Gordon, a retired teacher from the Toronto District School Board, was right when he said, “Literacy is the key to unlocking the world of Mathematics.” **Understanding Math Plus** © programs present many opportunities to explore literacy strategies. The **Fight Plan** model is an effective strategy in addressing all forms of communication.

The principal goal of a Flight Plan is to involve the students in meaningful discussions around the concepts explored in **Understanding Math Plus** ©. In the Flight Plan model each group of three students – group size may vary depending on student needs – is assigned a role in their Flight Crew.

The Flight Crew consists of three students. Each member of the crew has a specific role; Pilot, Co-pilot, or Navigator.

- a. **Pilot** is responsible for flying the plane – controlling the keyboard and mouse. Note that the pilot can only input instructions based on a group consensus.
- b. **Navigator** is responsible for recording their Flight Plan. She/he will record all the ideas and diagrams that their crew feels are essential to understanding a concept (see *Navigator’s report sheet*).
- c. **Co-pilot** is responsible for ensuring that all work is covered, building consensus for solutions, and on-task behavior.

The pre-flight briefing is an introduction by the teacher outlining where the crews will be flying. The flight is a metaphor for specific lessons within the **Understanding Math Plus** © programs. Usually a flight takes between 30 to 40 minutes. A teacher would have specific expectations in mind that they want their students to explore.

During the flight the crew is seated in front of the computer. The pilot is at the keyboard with their co-pilot on her/his right. The navigator sits slightly behind and in between the pilot and co-pilot. During the flight the navigator is recording, in point or sentence form, information that can be used to explain the concept. She/he is drawing figures, diagrams, etc. that help illustrate the concept(s). Once the flight is complete they return to their classroom for a whole-class debriefing.

The debriefing, facilitated by the teacher, is an opportunity to share and discuss the key points recorded during their flights. These discussions can be very rich as students work toward forming an understanding of a new concept. Each crew member then uses the Navigator’s Report to complete their journal entry about the concept.

Math Circles

With Understanding Math Plus Software©

The goal of this lesson strategy is to involve the students in a discussion around the concepts covered in the **Understanding Math©** software – the ‘Big Idea(s)’. In Math Circles students have the opportunity to participate in "math talks" in which they discuss what they're studying. By participating in Math Circles children have an opportunity to analyze what they've studied with a group of their peers. This discussion enhances their understanding and appreciation of the topic – the ‘Big Idea(s)’ e.g. *equivalent fractions mean the same thing but use different numbers in the numerator and denominator*. In addition to raising the level of student engagement, peer collaboration, and mathematics comprehension, Math Circles give students the opportunity to develop important time-management and cooperative group skills that will help them in future years.

Math Circle Format

Once the students have developed opportunities to study and learn about the topics through lessons taught using the Understanding Math Plus© programs (including journal writing, manipulative activities, worksheets and Word Walls) they meet as a Math Circle group. In their groups, they will write on chart paper, what they think is the big idea for this topic and posts them for a Gallery Walk. Class discussion around the posted ideas will occur with a consensus of the ‘Big Idea’ for this topic.

Each student in the circle will then complete one of the following roles during the circle. Note that for each topic, roles will be rotated. It is intended that all students will have the opportunity to complete each role as they work through math topics throughout the year.

Role completion will be assisted by each role participant, meeting with similar role participants from other groups. Role participants in their conferences will discuss the ‘Big Idea’ and how best to complete their role sheet. Once a consensus is reached, participants fill in their role sheet. Before returning to their Math Circle group, participants will practice how they are going to present their ideas (Role sheet) to the circle. This will ensure that all students will have the support they need to successfully complete and carry out their roles.

MATH CIRCLE ROLES

Discussion Leader

The Discussion Leader serves as the leader for the entire meeting, making sure that every student has a chance to participate. They will follow the “How to Lead a Discussion” instruction sheet. The Discussion Leaders also are responsible for coming up with the questions for the “Big Ideas”. This involves writing questions that incorporate higher-order thinking. The Discussion Leader uses the questions during the meeting to ensure understanding of the ‘Big Ideas’ of all participants. During the meeting the group will complete the questions and write the answers on the Discussion Leaders sheet for evaluation by the teacher.

Word Wizard (using Freyer Model)

The Word Wizards are responsible for analyzing unfamiliar or especially challenging words in the topic. The Word Wizards are to identify at least three difficult words, guess what they mean, and then actually look them up in the dictionary. The Word Wizards will include the section in the program where the words can be found so the members of the group can discuss the words in the context of the Understanding Math Plus program. They will then take their words and complete a Freyer Model activity for them, demonstrating a clear understanding for the words. In their Math Circle group the Word Wizards, while sharing their words, give the rest of the group time to share any words they found as they worked through this ‘Big Idea’.

Reporter

The Reporters are responsible for summarizing the ‘Big Ideas’. This can be difficult because the topic will often consist of several subtopics within the Understanding Math Plus© program. The Reporter has to summarize the ideas that lead up to the ‘Big Idea’ with a clear explanation, in words, of the ‘Big Idea’. The Reporter will need to make sure that the words clearly outline the understanding of the topic. After sharing the detailing of the ‘Big Idea’ in their Math Circles, the Reporter encourages group discussion and clarification if needed.

Connector

The Connectors are in charge of sharing the connections they have made as they analyzed the ‘Big Idea’. These could be Idea-to-Self, Idea-to-World, or Idea-to-Idea connections. After sharing their connections in their Math Circle, the Connectors will give the rest of the group time to share any connections they have made as they worked through their topic.

Illustrator

The Illustrators are responsible for illustrating the main ideas. This can be difficult because the topic will often consist of several subtopics within the **Understanding Math Plus©** program. The Illustrators have to illustrate the ‘Big Idea’. They will also need to make sure that their drawings are well labeled. After sharing their graphics in their Math Circle the Illustrators will give the rest of their

group time to share any additions or changes they feel need to be made to show understanding of their topic.

Note: Some members could have more than one role or share a role.

Materials Needed for Circle Meeting:

1. Role sheets (Discussion leader, Reporter, Connector, Illustrator, and Word Wizard)
2. Math Circles' Work Record Sheet
3. Journals
4. Self Evaluation sheet
5. Duo Tang folder
6. Additional aides
 - Sticky tabs
 - Students prepare questions they have on what they are not sure about, place them on sticky tabs and bring to circle for discussion by group.

The 'Big Idea' Prior to Math Circle formation:

- 1) Two to 3 students meet to go over the material that they have and form their interpretation of the 'Big Idea'.
 - a) Lessons done in the Understanding Math Plus© program,
 - b) Journal writing,
 - c) Manipulative activities,
 - d) Worksheets, and/or
 - e) Word Walls.
- 2) Gallery Walk
 - a) A 'Big Idea' is written on chart paper and posted on the wall for the Gallery Walk.
 - b) Groups walk and discuss what others have posted noting commonalities or differences
 - c) Class discusses findings - lead by the teacher - and finalizes the "Big Idea".
 - d) "Big Idea" is posted

Note for the teacher:

Correlation on website <http://www.neufeldmath.com/correlations> will help determine topic and subtopic for specific objectives covered in Math Circles.

Math Circle Classroom Procedure:

1. Groups of 4 students form Math Circles and decide on the assigned roles for each member. (see role sheets)
 - (a) Select Discussion Leader first
 - (b) Write name on role sheets

2. Math Circle group will then disperse to meet with corresponding role players from the other groups (expert groups).
 - (a) All students with similar role sheets will meet (i.e. All Connectors from each group on this topic will meet)

 - (b) All students with similar role sheets work together using the following resources:
 - i. Lessons done in the Understanding Math Plus© program,
 - ii. Journal writing,
 - iii. Manipulative activities,
 - iv. Worksheets,
 - v. Word Walls.
 - (ii) Discuss what is needed to complete their role – see role sheet.
 - (iii) Help each member to understand the ‘Big Idea’.
 - (iv) Check each members work.
 - (v) Practice presenting work before returning to Math Circle.

- (c) Time will be given for the members to work on their role sheet in class.

REMEMBER THAT THE COMPUTER WITH **UNDERSTANDING MATH PLUS©** IS A MEMBER OF EACH GROUP AND EACH GROUP IS ENCOURAGED TO REFER TO IT WHEN WORKING ON COMPLETING THEIR ROLES.

3. Math Circle will reform with the Discussion Leader calling the group to order; (see How to lead a Math Circle).

How to Lead a Math Circle

(Discussion Leader)

The following steps outline the steps a Discussion Leader would take in leading a 'Math Circle' meeting. Leaders must try to involve everyone in the discussion, and keep the group on task.

1. Reporter:

Ask the Reporter to summarize the 'Big Idea'. Discuss the summary. Allow other members to add details that may have been forgotten. Remember this is everyone's chance to help improve the report.

2. Illustrator:

Ask the Illustrator to present illustration for the 'Big Idea'. Discuss the illustrations. Allow other members to add details that may have been forgotten. Remember this is everyone's chance to help improve the report.

3. Word Wizard (Freyer Model)

Ask the Word Wizard to read a word and allow each member of the group to make an educated guess about the word. Ask the Word Wizard to give the dictionary definition and discuss its meaning. Word Wizard will show (Freyer Model) samples of what the word is and is not. Remember this is everyone's chance to help improve the report. Repeat this for each word.

4. Connector:

Ask the Connector to share the way they completed their answers. Allow group members to discuss the material briefly. Remember this is everyone's chance to help improve the report.

5. Discussion Leader:

Read aloud to the group the question and call on several people to respond to the questions. Have the group form and answer and allow members to add details. This answer will be marked by the teacher. Remember this is everyone's chance to help improve the report.

6. Work Report and Self-Evaluation:

Ask each member to make sure their work report and self-evaluation sheets are complete.

7. Wrap Up:

Ask each student to hand in their Math Circle sheet and staple them in order as follows.

*Group Rubric Sheet
Reporter's Sheet
Illustrator's Sheet
Word Wizard's Sheet
Connector's Sheet
Discussion Leader's Sheet
Each member's Work Record Sheet
Each member's Self-Evaluation Sheet*

If time permits ask everyone to make a prediction about what will happen next in this topic.

Evaluation/Assessment:

Teachers are encouraged to evaluate, through observation and checklist, students participating in Math Circles at **each** meeting.

This grade is based on (see *Teacher Class Observation - Daily Checklist*)

- Whether or not they are prepared on their meeting day.
 - They are expected to be done topic and subtopics assigned.
 - They must have their role sheet completed thoroughly.
 - They must have their Journal entries with them. (The students are to use their Journals throughout the meeting to discuss the topic so, each child must have their work with them)
 - They must also participate by making comments and asking questions throughout the meeting.

A traditional paper/pencil test covering the current Math Circle topic can be used for evaluation. This test could be only one question but demands higher-order thinking processes from the students. The question could be from a Math Circle group worksheet. You could also ask them to answer the test question using a variety of methods (multiple representations). Posting their answers - Gallery Walk - would help students to further deepen their understanding of the topic.

Remember, in addition to teacher evaluations, the students will be evaluating themselves as learners and contributors to the group. They must complete a "Self-Evaluation" sheet at the end of each Math Circle meeting.

Discussion Leader:

Name: _____

You are responsible for coming up with the question(s) for the 'Big Idea'. Write the question(s) so that it incorporates the 'Big Idea'. In your role group, discuss what the correct answer(s) should look like. You will use the question(s) during the meeting to encourage discussion among the members of the group. The Math Circle group will answer the question(s) for marking by the teacher. This question(s) should not limit discussion. The Discussion Leader serves as the leader for the entire meeting and makes sure that every student has a chance to participate.

Date: _____

Big Idea: _____

Question:

Answer: May incorporate symbols, illustrations, words, etc.

How to Lead a Math Circle

(Discussion Leader)

Use the following steps to help you lead today's 'Math Circle' meeting. Try to involve everyone in the discussion, and remember to keep the group on task.

1. Reporter:

Ask the Reporter to summarize the 'Big Idea'. Discuss the summary. Allow other members to add details that may have been forgotten. Remember this is everyone's chance to help improve the report.

2. Illustrator:

Ask the Illustrator to present illustration for the 'Big Idea'. Discuss the illustrations. Allow other members to add details that may have been forgotten. Remember this is everyone's chance to help improve the report.

3. Word Wizard (Freyer Model):

Ask the Word Wizard to read a word and allow each member of the group to make an educated guess about the word. Ask the Word Wizard to give the dictionary definition and discuss its meaning. The Word Wizard will show (Freyer Model) samples of what the word is and is not. Remember this is everyone's chance to help improve the report. Repeat this for each word.

4. Connector:

Ask the Connector to share the way they completed their answers. Allow group members to discuss the material briefly. Remember this is everyone's chance to help improve the report.

5. Discussion Leader:

Read aloud to the group the questions and call on several people to respond to the questions. Read Have the group from and answer and allow members to add details. Remember this is everyone's chance to help improve the report.

6. Work Report and Self-Evaluation:

Ask each member to make sure their work report and self-evaluation sheets are complete.

7. Wrap Up:

. Ask each student to hand in their Math Circle sheet and staple them in order as follows.

Reporter's Sheet

Illustrator's Sheet

Word Wizard's Sheet

Connector's Sheet

Discussion Leader's Sheet

Each member's Work Record Sheet

Each member's Self-Evaluation Sheet

If time permits ask everyone to make a prediction about what will happen next in this topic. Hand in Circle Sheets and collect next topic from the teacher. Make sure that everyone knows what the next topic assignment is for the next week.

ILLUSTRATOR:

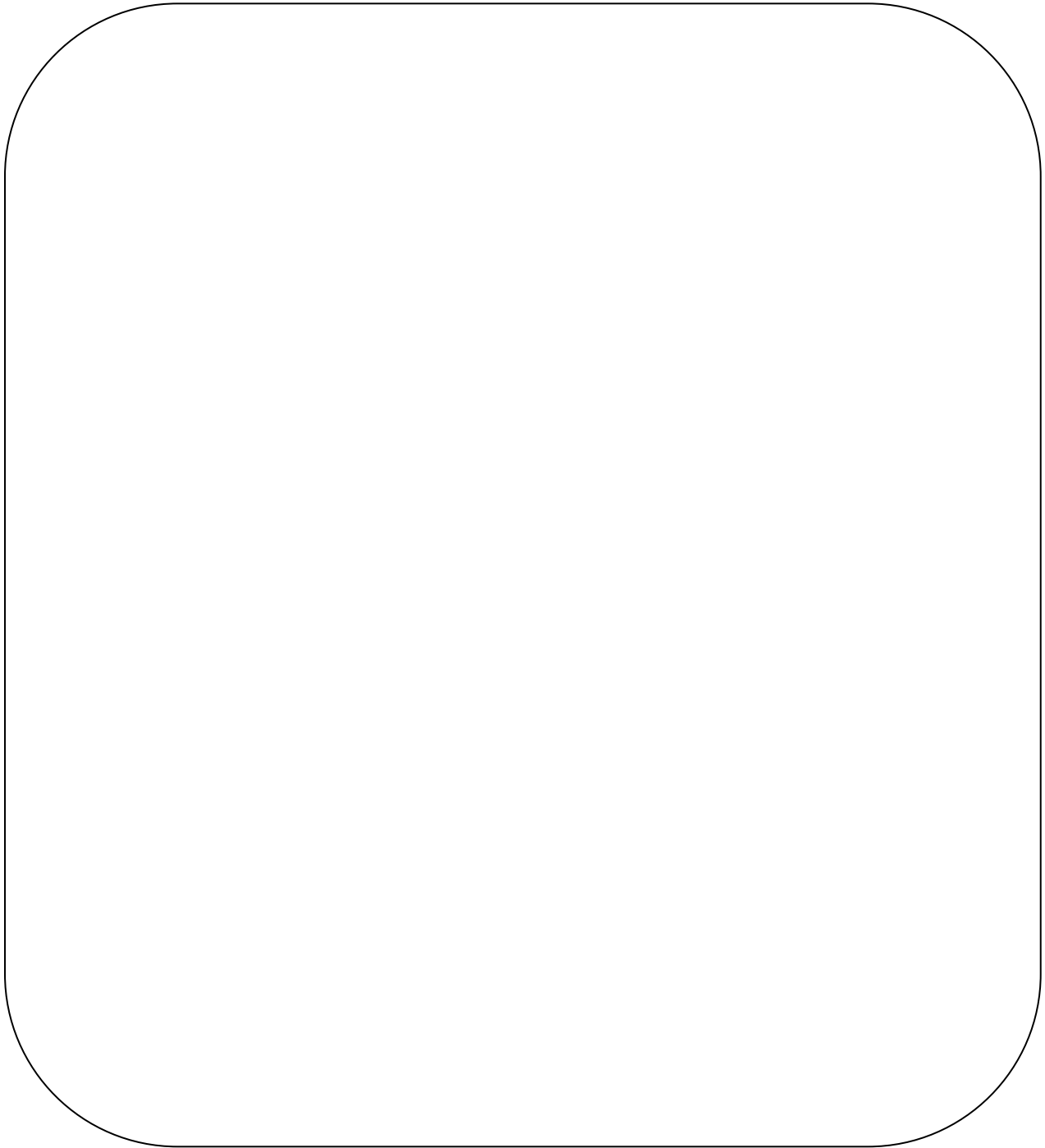
Name: _____

You are responsible for illustrating the 'Big Idea'. This can be difficult because the topic will often consist of several subtopics. The Illustrator has to illustrate 'Big Idea'. You will need to make sure that the illustrations are well labeled.

Date: _____

'Big Idea': _____

Illustrating the 'Big Idea': (clearly label all parts)

A large, empty rounded rectangular box with a thin black border, intended for the student to draw an illustration of the 'Big Idea'.

Word Wizard:

Name: _____

You are responsible for analyzing unfamiliar or especially challenging words in the 'Big Idea'. You are to identify three to five difficult words, guess what they mean, and then actually look them up in the dictionary. Notice, you will need a Freyer Model sheet for each word – see sample below. Within the Freyer Model sheets, you need to include examples of what the word is and what the word is not. Notice illustrations and labeling will add to your examples. In the Math Circle, while sharing your words, it will give the rest of the group time to share any words they found as they worked through the 'Big Idea'

Date: _____

Big Idea: _____

My Understanding of the word	Definition
The Word	
Examples	Non examples

Other Group Strategies

In order for students to become competent in collaborative situations they will need ample opportunities to apply their co-operative group skills along with teacher guidance. Students **will** learn co-operative skills through the repeated cycle of experience (actively “doing it”), reflection, understanding, and practice. Through reflecting on their own experience students discover the need to improve some aspect of their group’s functioning or their own functioning within the group. They learn about the causes of difficulties in the groups. They create and try out their own solutions. Within this learning cycle of *experience-reflection-understanding-practice* they experience the satisfaction of learning a new skill or overcoming challenges.

(Adapted from; Clarke, J., Wideman, R., Eadie, S., “*Together We Learn*”, Prentice Hall Canada Inc., Scarborough, Ontario, 1990.)

“Students who talk and listen to one another converse about mathematical concepts gain experience in reflecting, in reasoning, and in developing mathematical language. There is no doubt that *student talk* is one of the most effective ways for students to demonstrate and to improve their understanding.”

(“*A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6: Volume Two, Problem Solving and Communication*”, Ontario Ministry of Education, Queen’s Printer for Ontario, 2006.)

Think/Pair/Share

This is an ideal strategy that encourages oral communication among students. Students individually consider an issue or problem and then discuss their ideas with a partner. Teachers may want to monitor the amount of time each student has to discuss their ideas/perspectives.

Fish Bowling

This technique allows **one** member from a group to circulate throughout the room observing groups engaged in a similar task. The observer is not allowed to ask any questions nor comment about what they observe – they must watch in silence. They then return to their home group and report about they have seen. They may or may not incorporate ideas from other groups.

Orbits

This technique is ideal for promoting dialogue and student ownership of ideas/processes. In a group of 3 to 4 students each is given a number or letter designation e.g. A, B, C, or D. Orbits should be used *after* a collaborative activity is completed. The students are told that one member of the group, chosen randomly, will be selected to stay (e.g. B) at their home group while the others (e.g. A, C, and D) are asked to rotate either clockwise or counter-clockwise to the next group. After the rotation is complete the group member who “stayed home” must explain to her/his new members the process/solution their group used to solve their activity. This pattern continues until every letter has had a chance to “stay home”. It is amazing to watch how students take ownership of ideas, even those ideas that aren’t their own. As the rotation pattern continues students will be asked to “stay home” at groups which were not their original home group. This definitely places an emphasis on students to pay attention to the presentation at each group they visit.

Gallery Walk

This technique also promotes oral communication in the classroom. After groups have completed their assigned activity on chart paper they are asked to post their work in the classroom. A collection of group work is on display in a location of the classroom and students are free to browse through the work of other groups and ask questions about their solution(s). The teacher's role is to facilitate and model effective and constructive questioning of student work e.g. *"Why did your group use a bar graph in your solution?"*


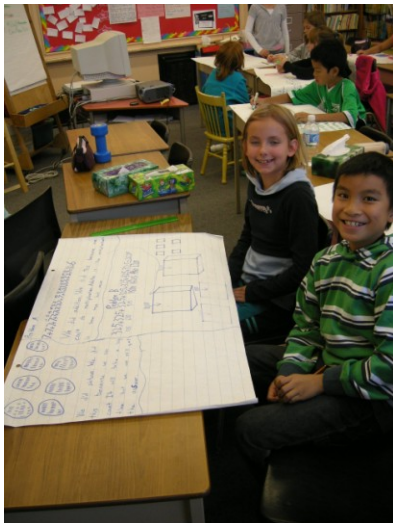

Bansho Method

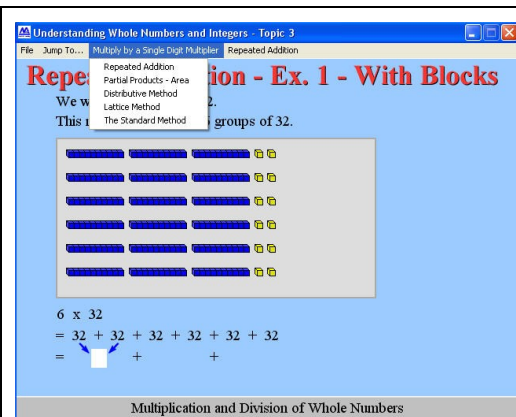
The Bansho Method has been adapted from the "Bansho" technique used in Japanese classrooms. Japanese teachers try to keep all that is written or drawn during a lesson on the chalkboard. In this way it is easier to compare multiple solutions to a single problem if they appear simultaneously on the chalkboard – a "birds-eye view" of the lesson.

We have adapted the Bansho technique as an extension of a Gallery Walk where students are asked to examine all group solutions looking for similarities between them. Students are actively engaged in this process and the teacher acts as a facilitator supporting student discussion and questioning. Similar solutions are grouped together to further emphasize the multiple solutions to a problem.

"It's never too late to start the collaborative process in your classroom."

**PICTORAL EXEMPLARS
GALLERY WALK / BANSHO TECHNIQUE
and
WHOLE CLASS LESSON**

PICTURE	DESCRIPTION
GALLERY WALK / BANSHO TECHNIQUE	
	<p>Students were given a problem to solve that involved multiplication e.g. 7×18. The teacher gave them two such problems; one was more challenging than the other. Students were asked to solve the problem in at least two different ways but they <i>could not</i> use the multiplication algorithm. They were encouraged to build, draw and write their solution.</p>
	<p>Students were given Base 10 blocks, Cubi-Links, and tape measures for their concrete manipulatives.</p> <p>In this picture you can see examples of grouping methods, repeated addition, use of Base 10 materials and written explanations.</p> <p>Groups were heterogeneous based on math ability.</p>
	<p>This is an example of a Gallery Walk display. These students are presenting their work and one has had <i>a lot</i> of chocolate!</p> <p>Students are able to see ALL of their classmate's solutions.</p> <p>The Bansho technique involved the teacher as a facilitator and together with her students they actively looked for solutions that were similar.</p>



The next lesson (whole class) was supported with Understanding Whole Numbers and Integers **Topic 3: Multiplying and Dividing Whole Numbers**. The lesson is titled “*Multiply by a Single Digit Multiplier*”.

The teachers had students use Base 10 blocks while working along with the software. The downloaded worksheet from www.neufeldmath.com was modified to suit the teacher’s needs.

WHOLE CLASS LESSON (Grade 2/3)



The lesson began with a review of pictographs and the basic elements of a good graph – titles, labels and legend.

Students were divided into mixed ability groups of two. Each group was given a bag of candy with different combinations of candy types/brands.

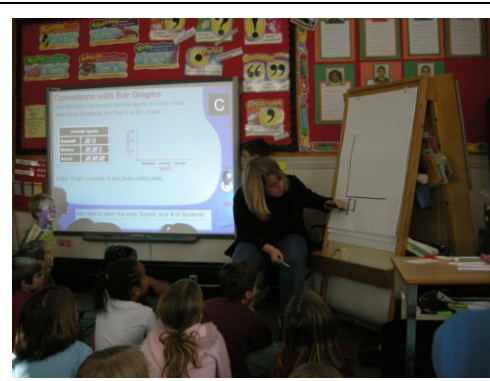
This took 5 to 10 minutes.



Students returned to their desks and began the sorting process. They gave their concrete pictograph a title, legend and appropriate labels.

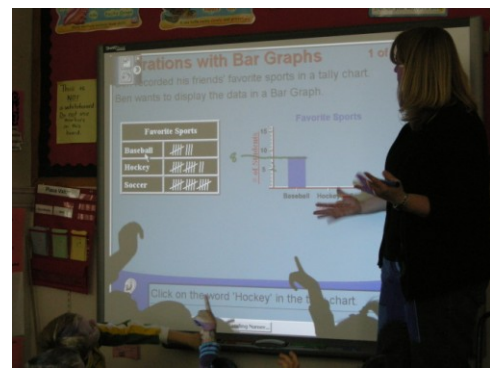


Note the concrete representation of a pictograph.



The teacher brought the class back together to extend their knowledge of pictographs **to** bar graphs.

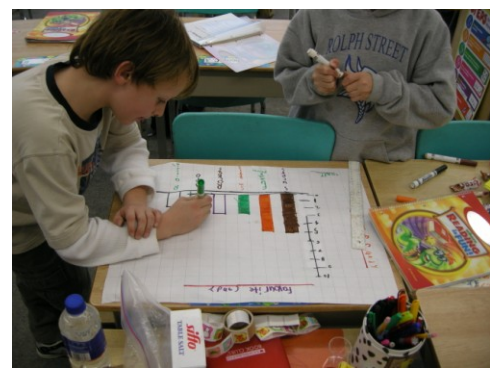
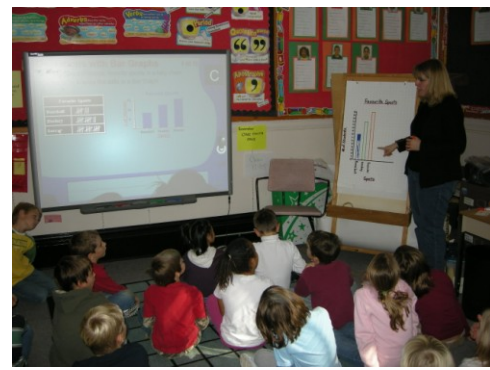
She used **Understanding Numeration: Concept** – Operations, **Skill** - #39 “Given Graph . . . Perform Operations”, **Level** – C, **Lesson** – Operations with Bar Graphs.



The teacher controlled the pacing of the lesson. She also used students to control the SmartBoard® and input answers.

She was able to model the construction of the same bar graph alongside the software.

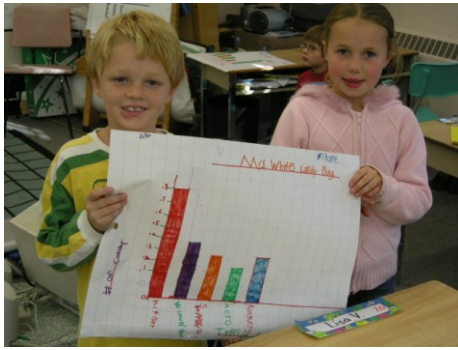
All the essential elements of a well constructed graph were emphasized i.e. title, labels and scales.



The conversion of their pictograph to a bar graph in process.



It was easier for the children to take the *concrete representation* of the pictograph and convert it to an *abstract representation* in a bar graph.



Proud mathematicians with their finished product.



Another excellent creation!

Observations:

- Students (primary level) were completely engaged during the double period of math
- Students were not frustrated during the extension phase of the lesson – pictograph to bar graph because of;
 - Heterogeneous groups
 - Teacher created “bridge” from concrete to abstract
- Activity was meaningful. This was the second day after Halloween.
- Students were always clear about what was expected.